

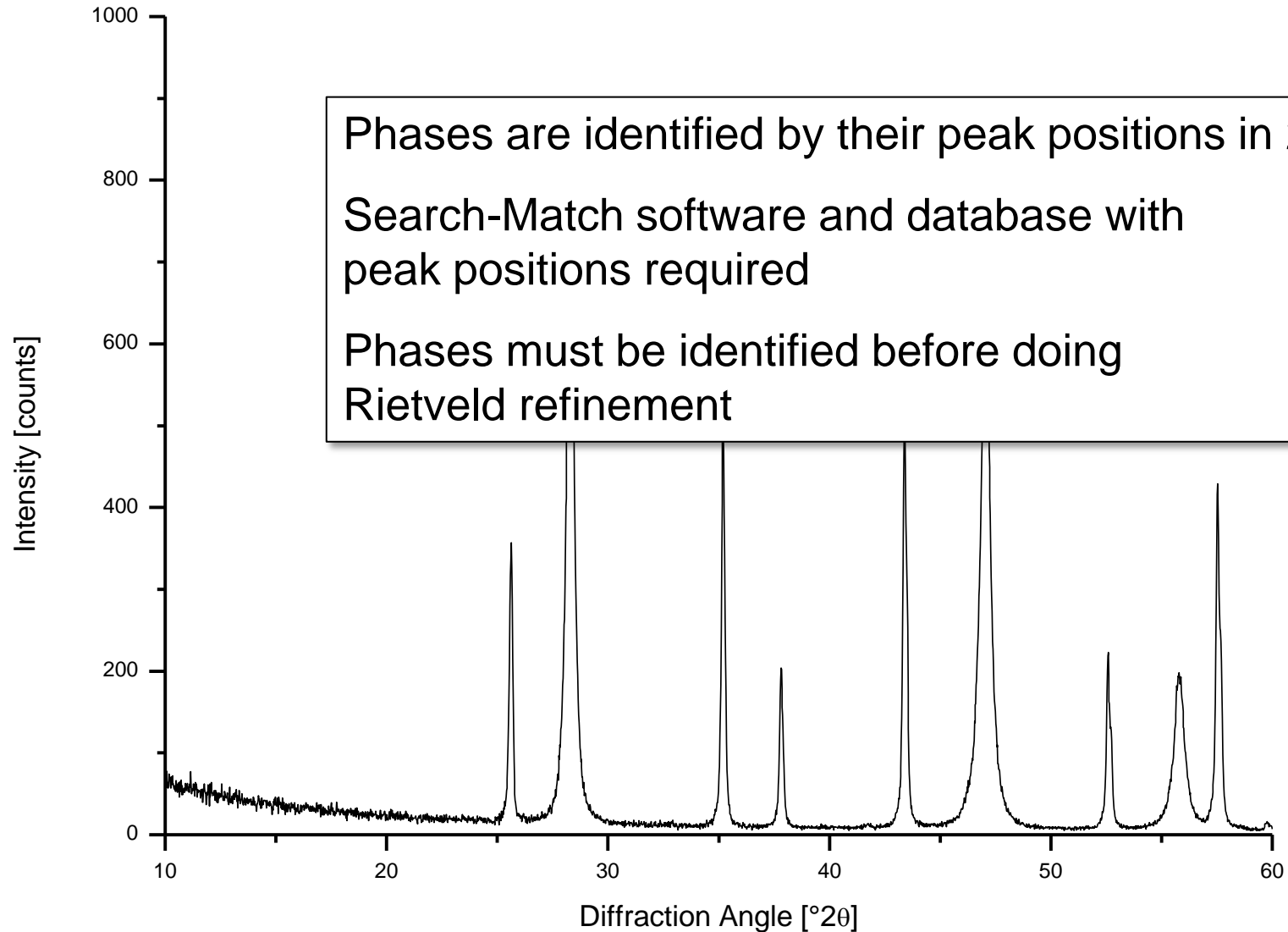
Lesson 5

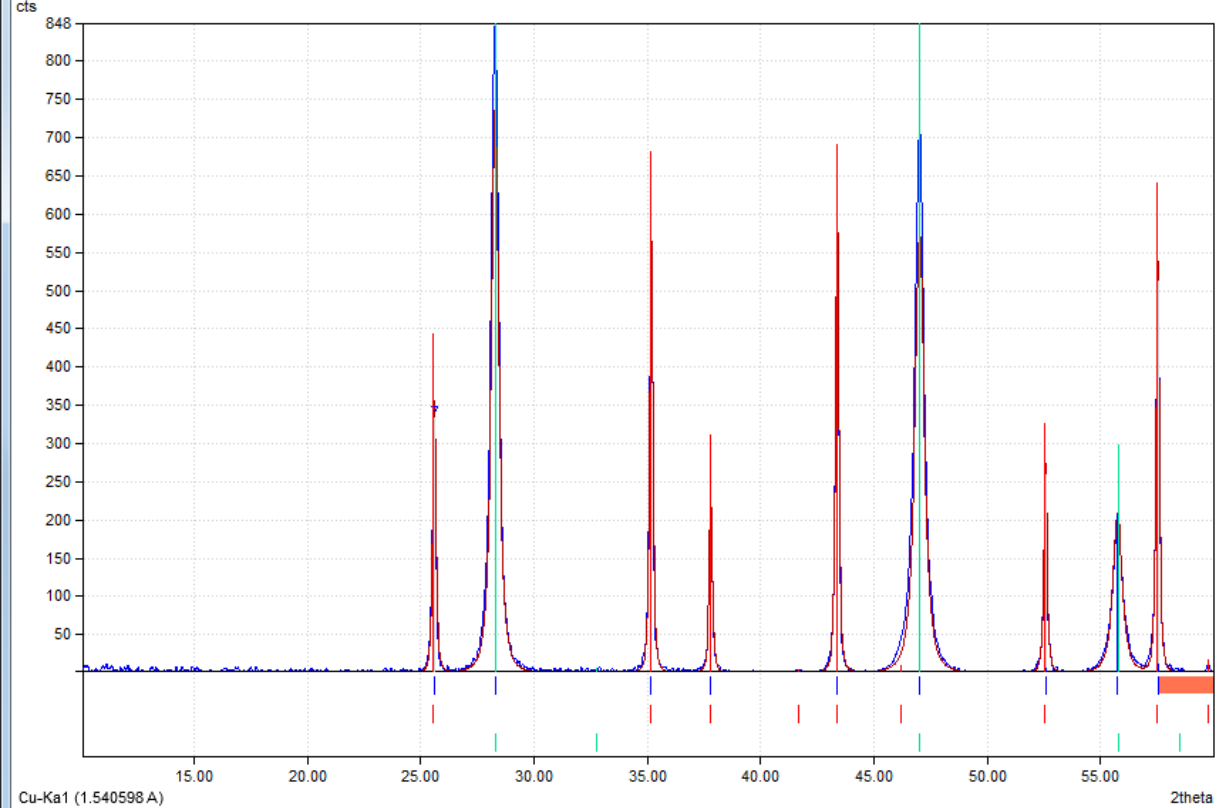
Rietveld Refinement

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Repetition: Phase Identification





2theta	Intensity	FWHM	Corundum, syn	Fluorite, syn
25.61	333.2	0.1638	442.4	
28.29	843.1	0.3740		981.9
32.78				5.0
35.18	530.3	0.1607	681.2	
37.80	215.5	0.1636	310.8	
41.67			4.2	
43.38	567.0	0.1633	691.7	
46.17			9.7	
47.02	713.1	0.4800		1000.0
52.57	258.5	0.1693	325.4	
55.75	207.0	0.4800		298.2
57.52	511.4	0.1688	641.1	
58.51				3.0
59.73			16.6	
61.12			22.1	
61.30			49.1	
66.51			239.5	
68.20			373.9	
68.71				102.1
70.41			8.3	
74.29			8.3	
75.90				86.0
76.87			100.4	
77.23			60.2	
78.24				6.0
80.41			5.5	
80.68			35.3	
83.20			2.1	
84.34			34.6	
85.12			2.1	
86.34			23.5	
86.49			17.3	
87.44				151.1

Color	Qual.	Entry	Formula	Candidate phase	P(2theta)	P(I/I0)	I scale fct.	I/Ic	FoM
I		01-077-2248	(Ca F2)0.75 (Y F3...	Calcium Yttrium Fluoride	0.9784	0.7850	1.0000	3.59	0.9119
*		00-004-0864	Ca F2	Calcium Fluoride (Fluorite, syn)	0.9893	0.6180	1.0000	2.40	0.8990
I		04-013-7404	Na0.50 Y0.50 F2	Sodium Yttrium Fluoride	0.9945	0.3934	1.0000	4.93	0.8783
I		01-074-5823	(Fe0.065 Ga0.63) ...	Iron Gallium Selenide	0.9636	0.0731	1.0000	9.06	0.8501
I		04-004-7480	Ga0.67 Se	Gallium Selenium	0.9636	0.0732	1.0000	9.03	0.8501
I		04-003-9968	Ga0.5 Ge0.13 Se	Gallium Germanium Selenium	0.9636	0.0708	1.0000	8.95	0.8498
B		04-014-0211	Si	Silicon	0.9974	0.0784	1.0000	4.55	0.8487
I		04-003-2846	AlP	Aluminum Phosphide	0.9973	0.0711	1.0000	4.50	0.8480
I		04-006-2647	Ga Sb0.1 P0.9	Gallium Antimony Phosphide	0.9945	0.0765	1.0000	8.46	0.8477
I		04-008-0619	Cd0.855 La0.095 F2	Cadmium Lanthanum Fluoride	0.9636	0.0652	1.0000	11.87	0.8467
B		04-012-6328	Zn S	Zinc Sulfide (Sphalerite)	0.9932	0.0575	1.0000	8.41	0.8453

Color	Entry	Formula	Matched phase	Quant.(%)
■	01-089-7717	Al2O3	Corundum, syn	56.4
■	04-002-2204	Ca F2	Fluorite, syn	43.6

For more than just identification:
Rietveld refinement



Prof. Hugo Rietveld

Extracts much more information from powder XRD data:

- Unit cell dimensions
- Phase quantities
- Crystallite sizes / shapes
- Atomic coordinates / Bond lengths
- Micro-strain in crystal lattice
- Texture effects
- Substitutions / Vacancies

No phase identification!

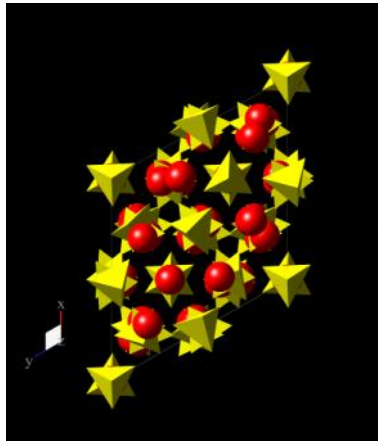
Identify your phases first
(unknown phase → no Rietveld refinement)

No structure solution
(just structure refinement)

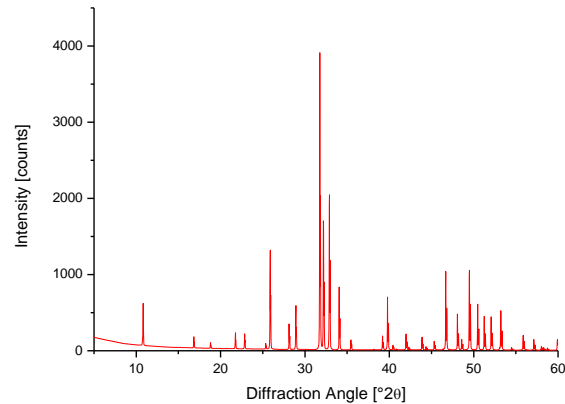
Needs excellent data quality!

Rietveld Refinement

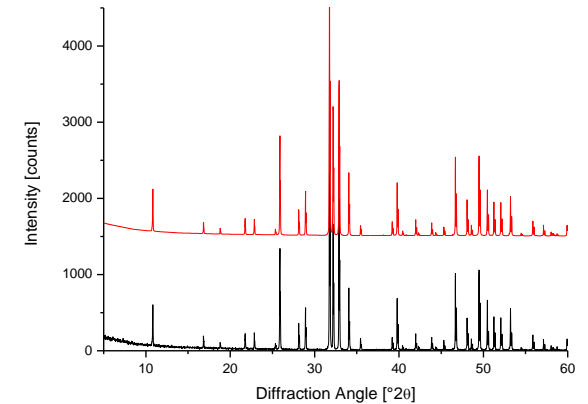
Known structure model



Calculate theoretical diffraction pattern



Compare with measured pattern



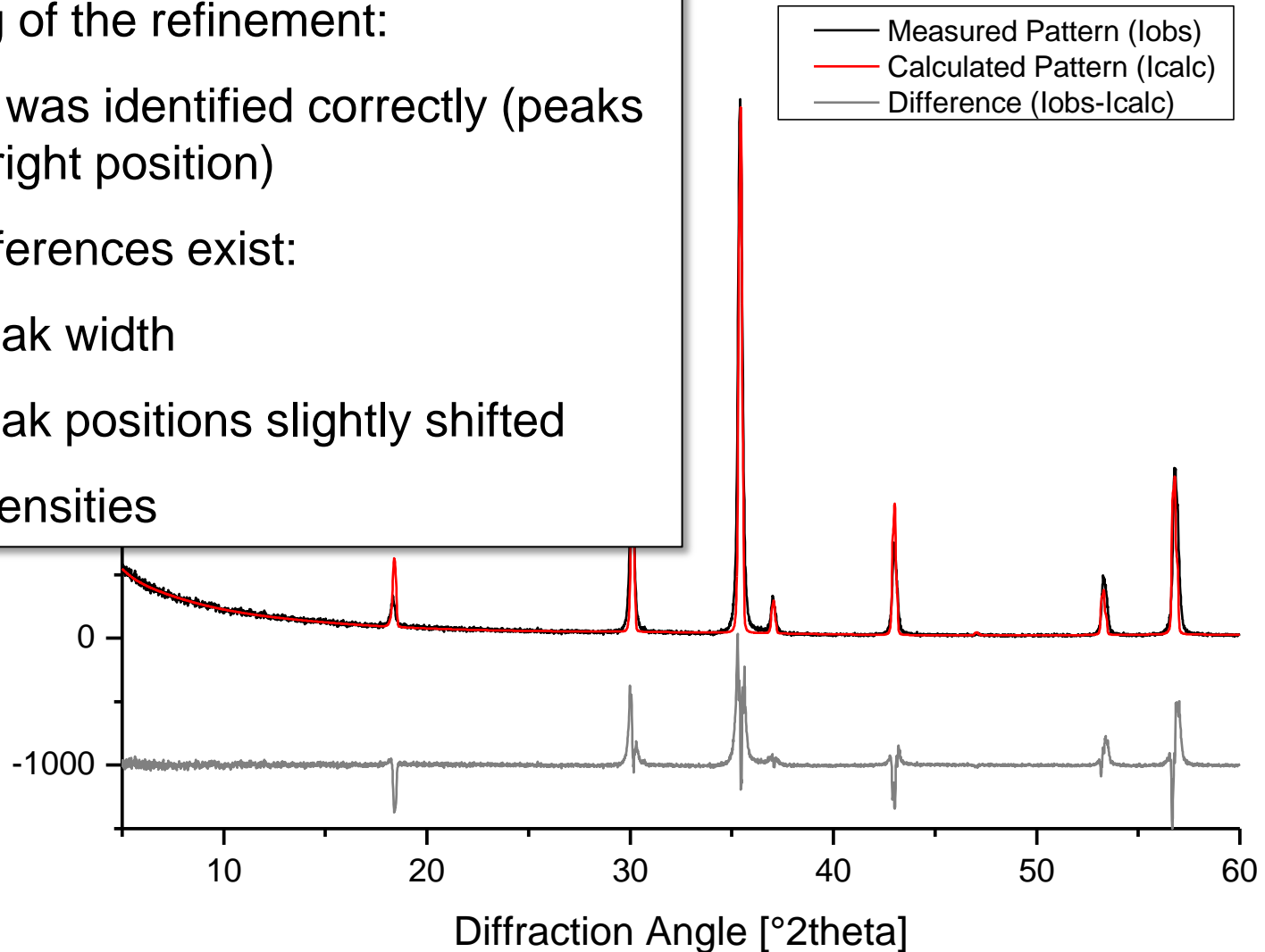
Optimize structure model, repeat calculation

Minimize differences between calculated and observed pattern by least-squares method

Rietveld Refinement

Beginning of the refinement:

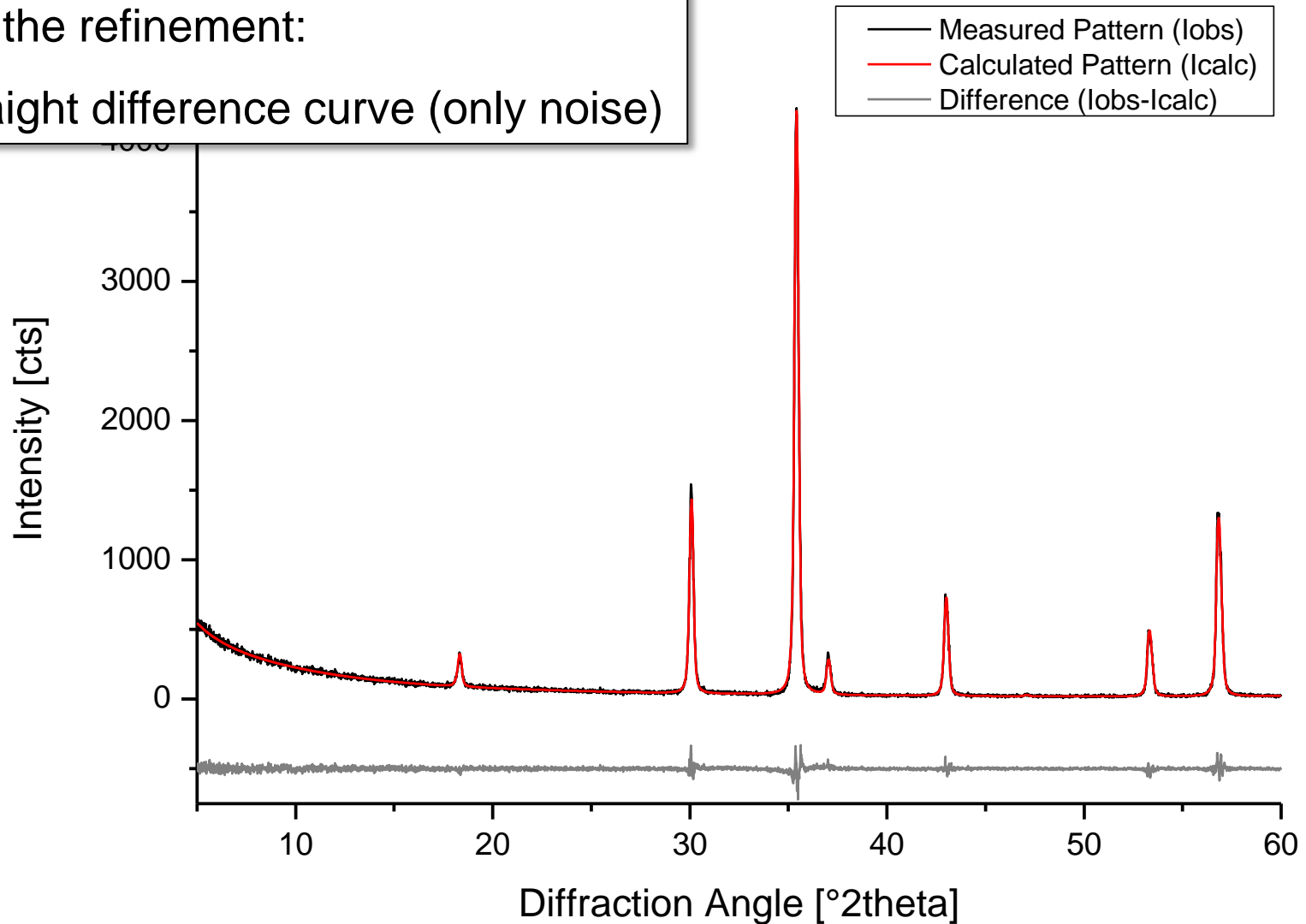
- Phase was identified correctly (peaks at the right position)
- But differences exist:
 - Peak width
 - Peak positions slightly shifted
 - Intensities



Rietveld Refinement

After the refinement:

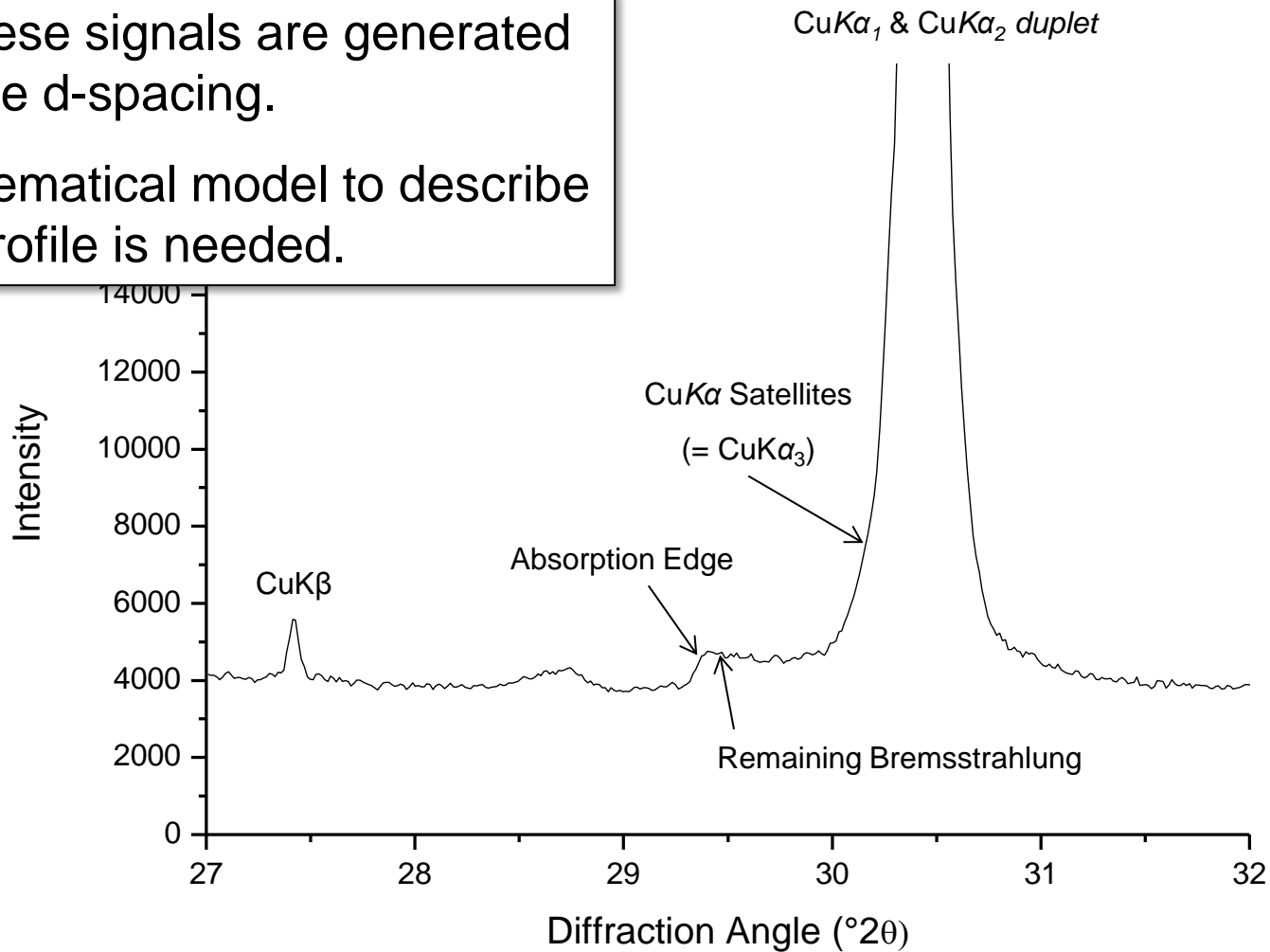
- Straight difference curve (only noise)



Modelling the Peak Profile

All these signals are generated by one d-spacing.

Mathematical model to describe the profile is needed.



Modelling the Peak Profile

Traditional («Rietveld») Approach:

Pseudo Voigt curves for $K\alpha_1$, $K\alpha_2$ and $K\beta$

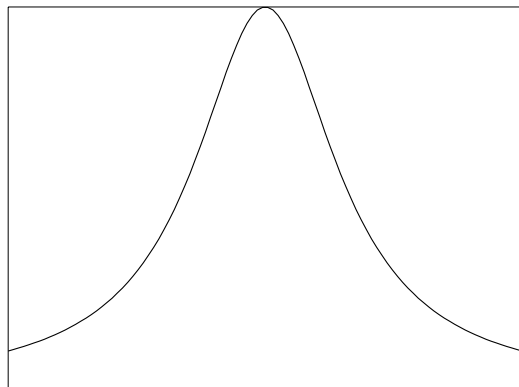
$$V_P(x) = n * L(x) + (1-n) * G(x)$$

Lorentzian curve

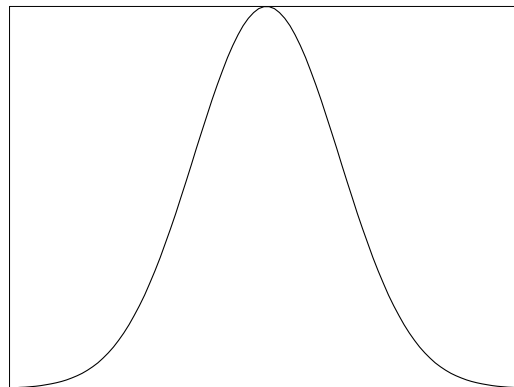
Gaussian curve

$$L(x) = \frac{1}{1 + \left(\frac{x-x_0}{\omega}\right)^2}$$

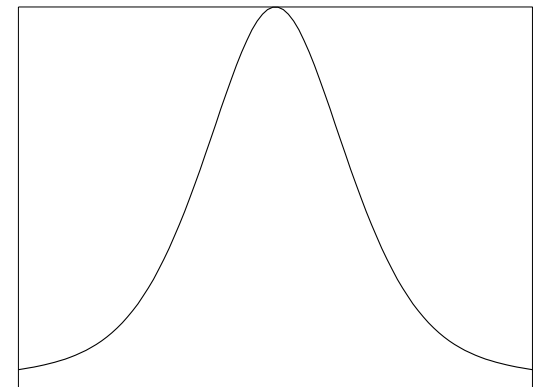
$$G(x) = \exp[-\ln(2) \cdot \left(\frac{x-x_0}{\omega}\right)^2]$$



Lorentzian ($\omega = 1.0$)

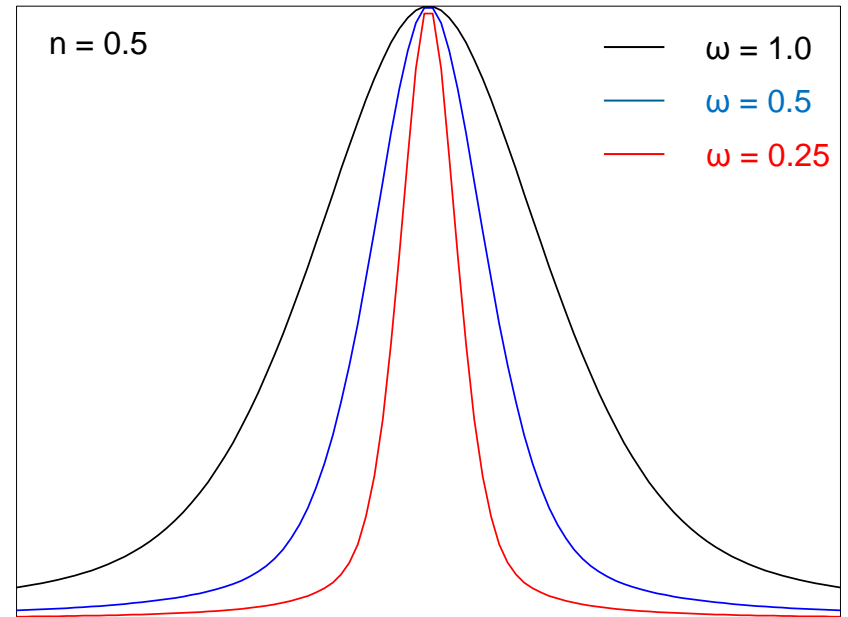
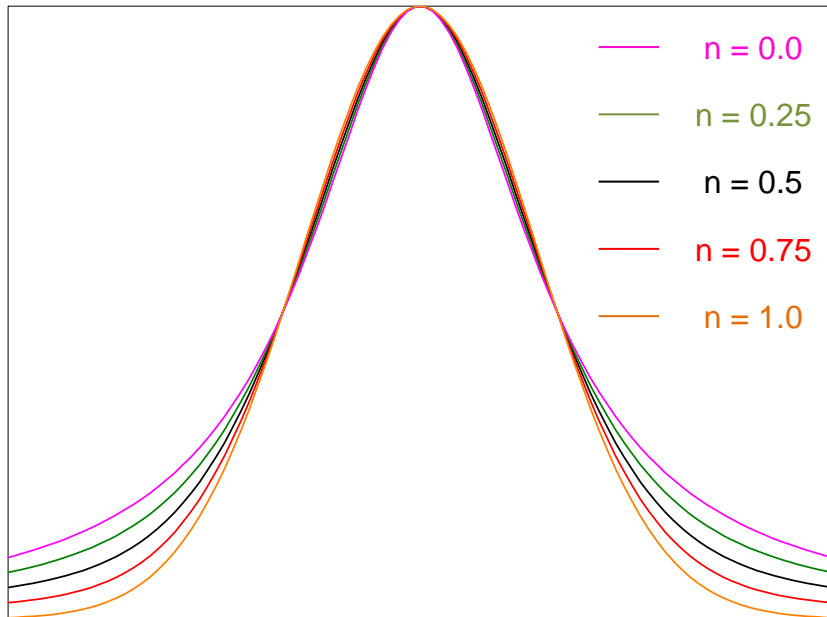


Gaussian ($\omega = 1.0$)

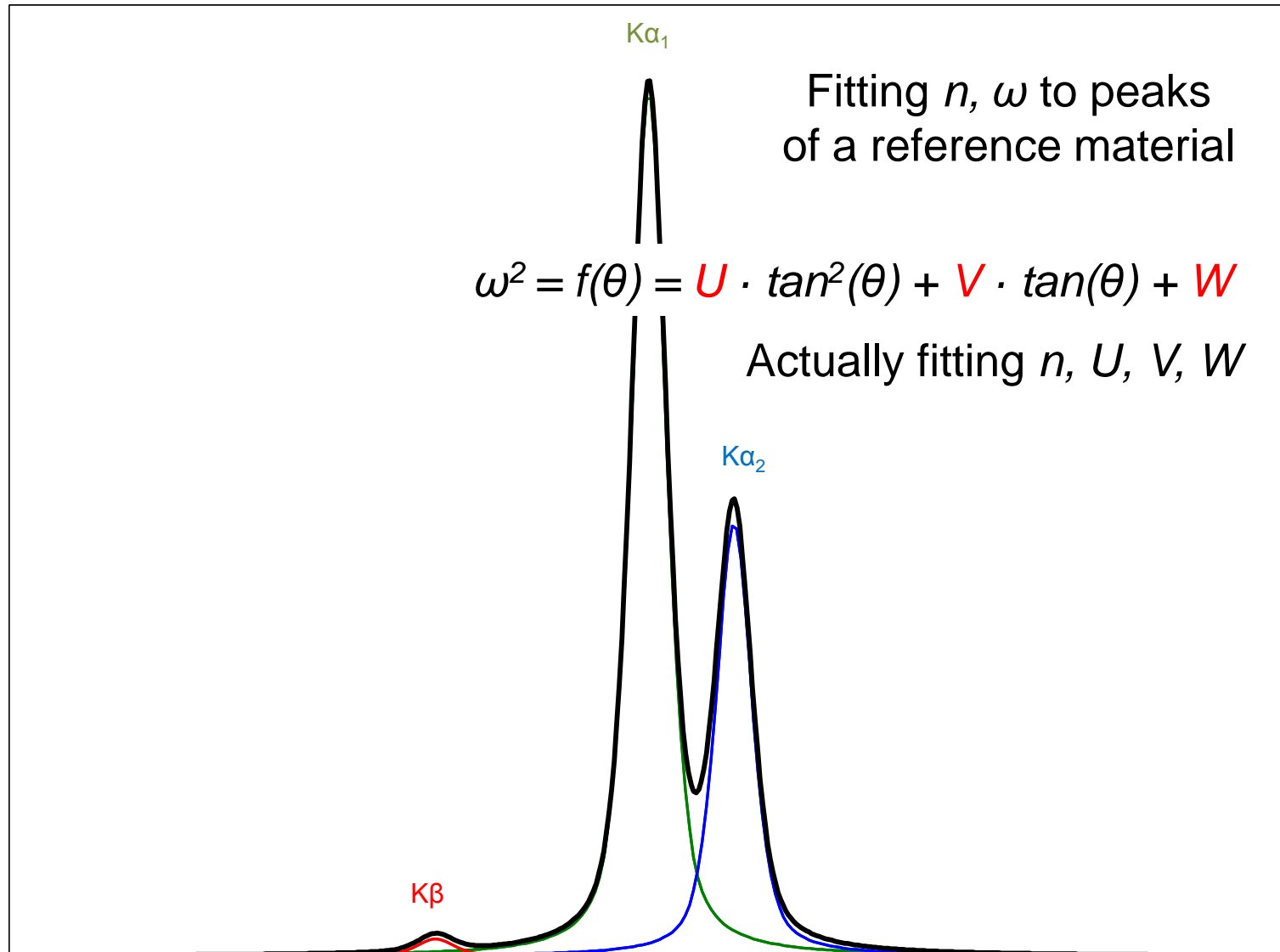


Pseudo-Voigt ($n = 0.5$)

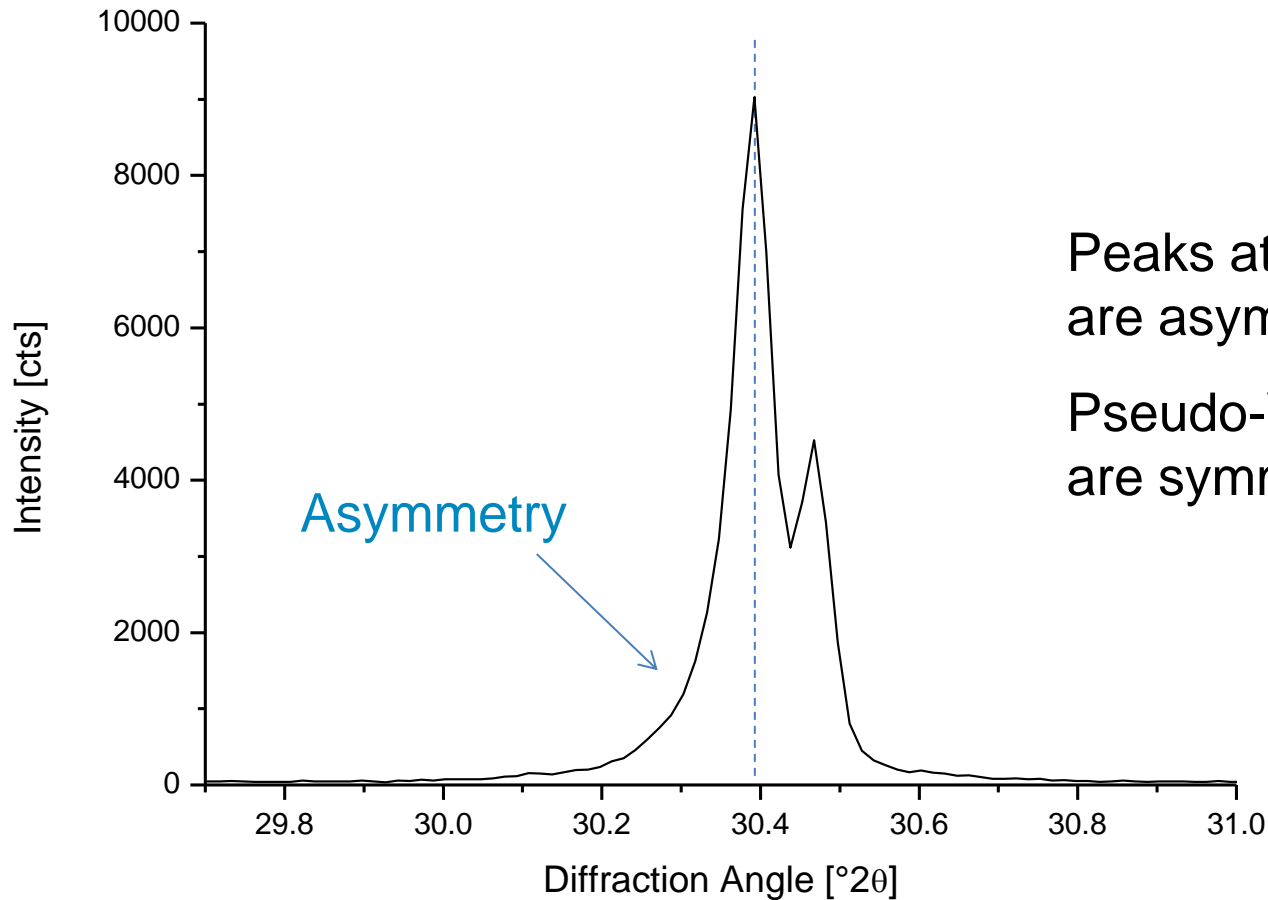
Pseudo-Voigt Curves



Pseudo-Voigt Curves



Pseudo-Voigt: Problems



Peaks at low 2θ angles are asymmetric.

Pseudo-Voigt curves are symmetric.

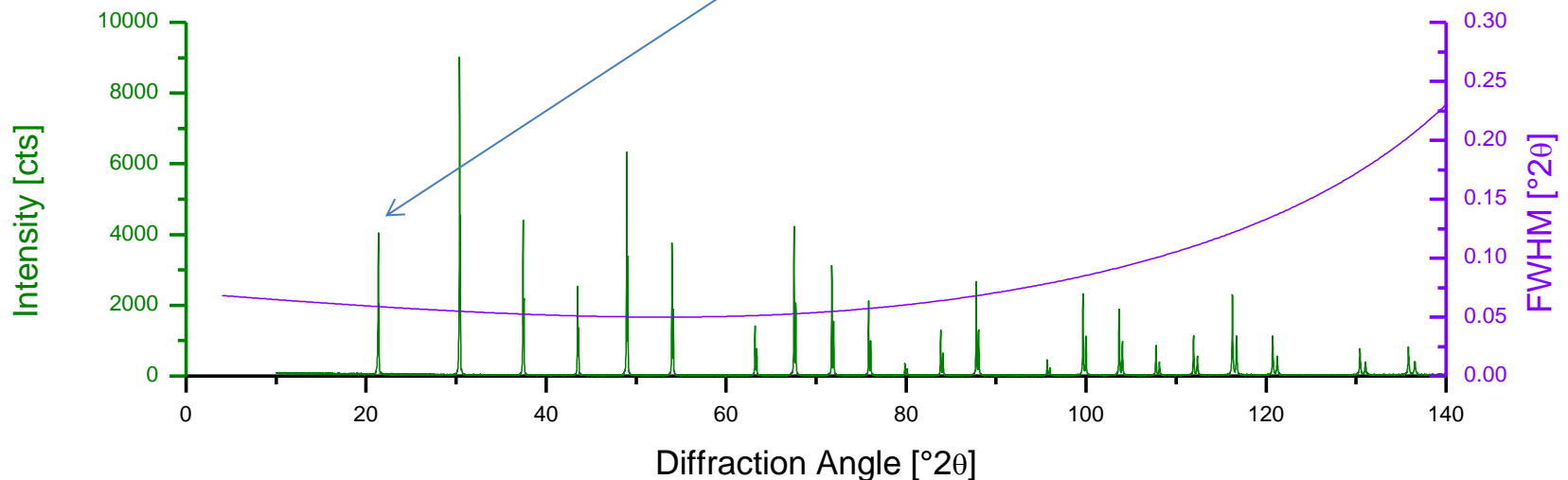
Alternatives to Pseudo-Voigt Function

Alternatives to PV function:

- Pearson VII
- Thompson-Cox-Hastings PV
- Split PV
- PV with axial divergence (Finger-Cox-Jephcoat PV)

FWHM of all these functions must be fitted to peaks of a reference material.

Common reference materials do not have peaks $< 20^\circ 2\theta$

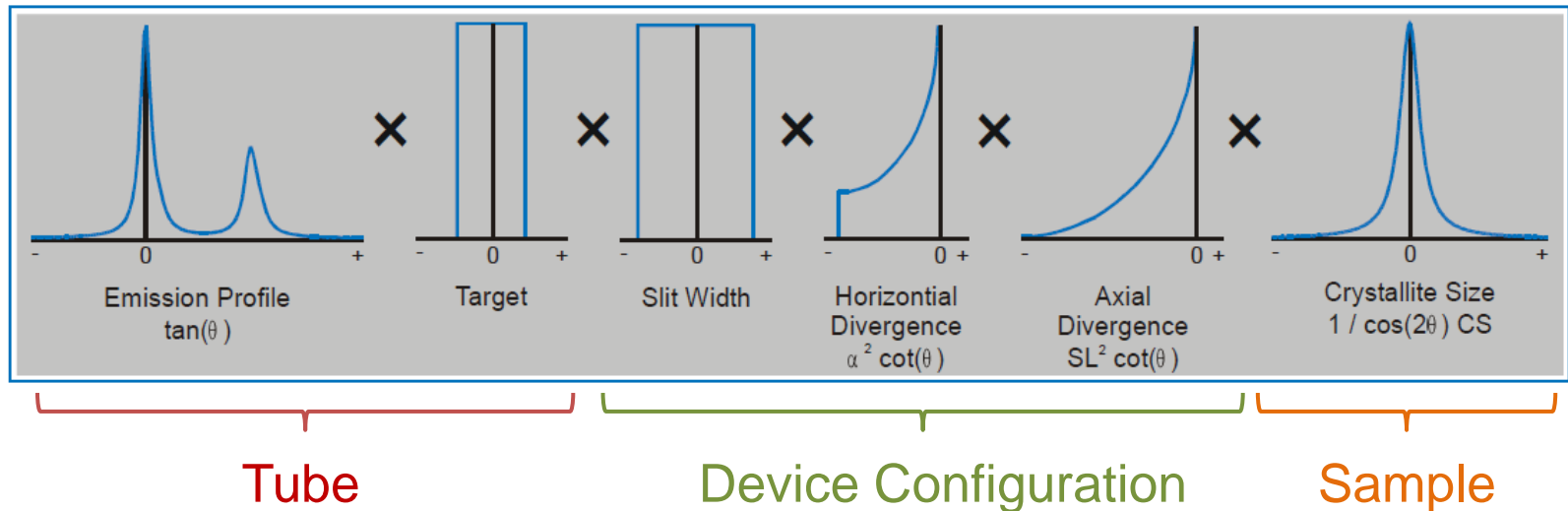


Fundamental Parameters Approach FPA

Calculate the peak profile from the device configuration

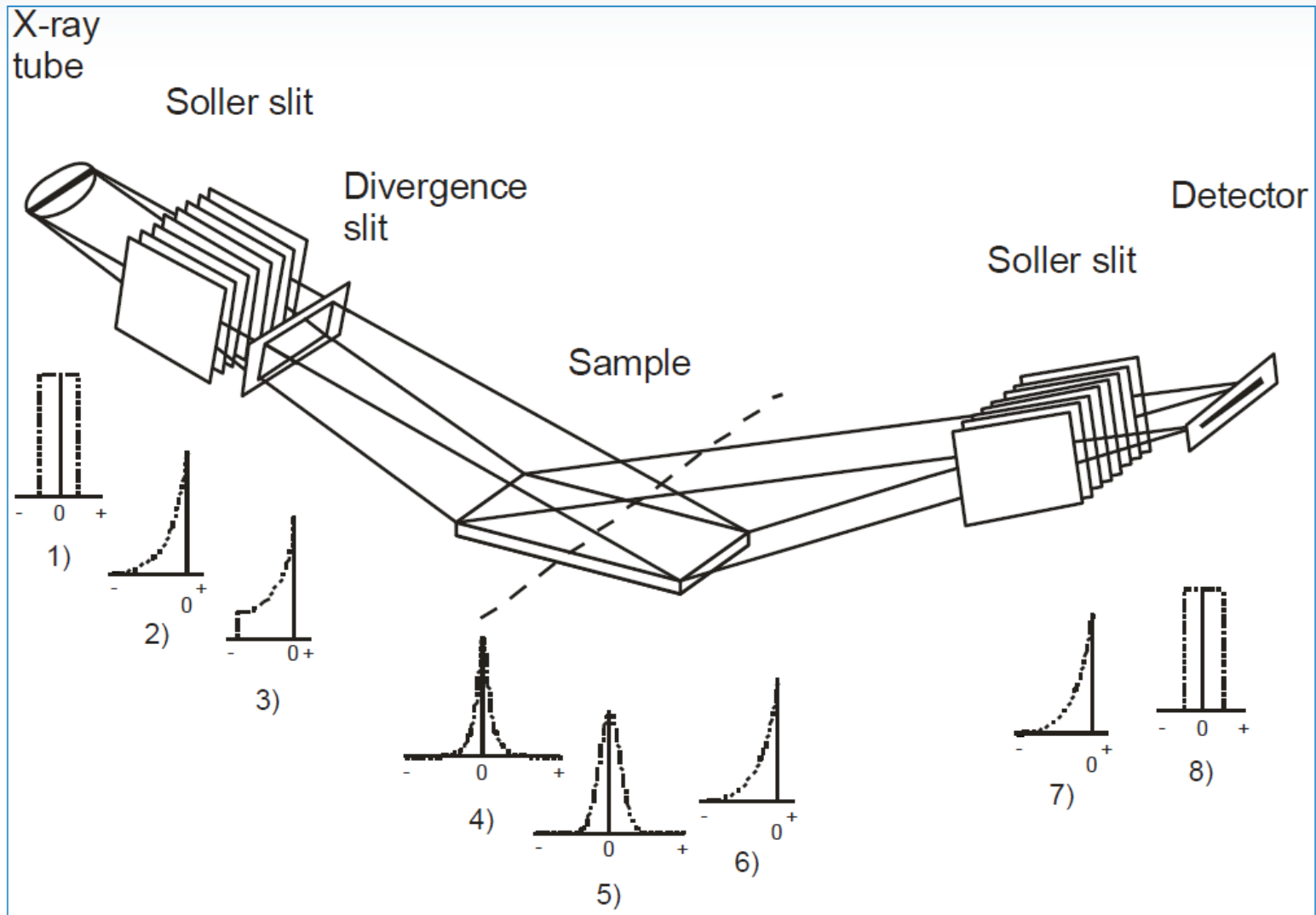
Take into account the contributions of:

- Source emission profile (X-ray wavelength distribution from Tube)
- Every optical element in the beam path (position, size, etc.)
- Sample contributions (peak broadening due to crystallite size & strain)



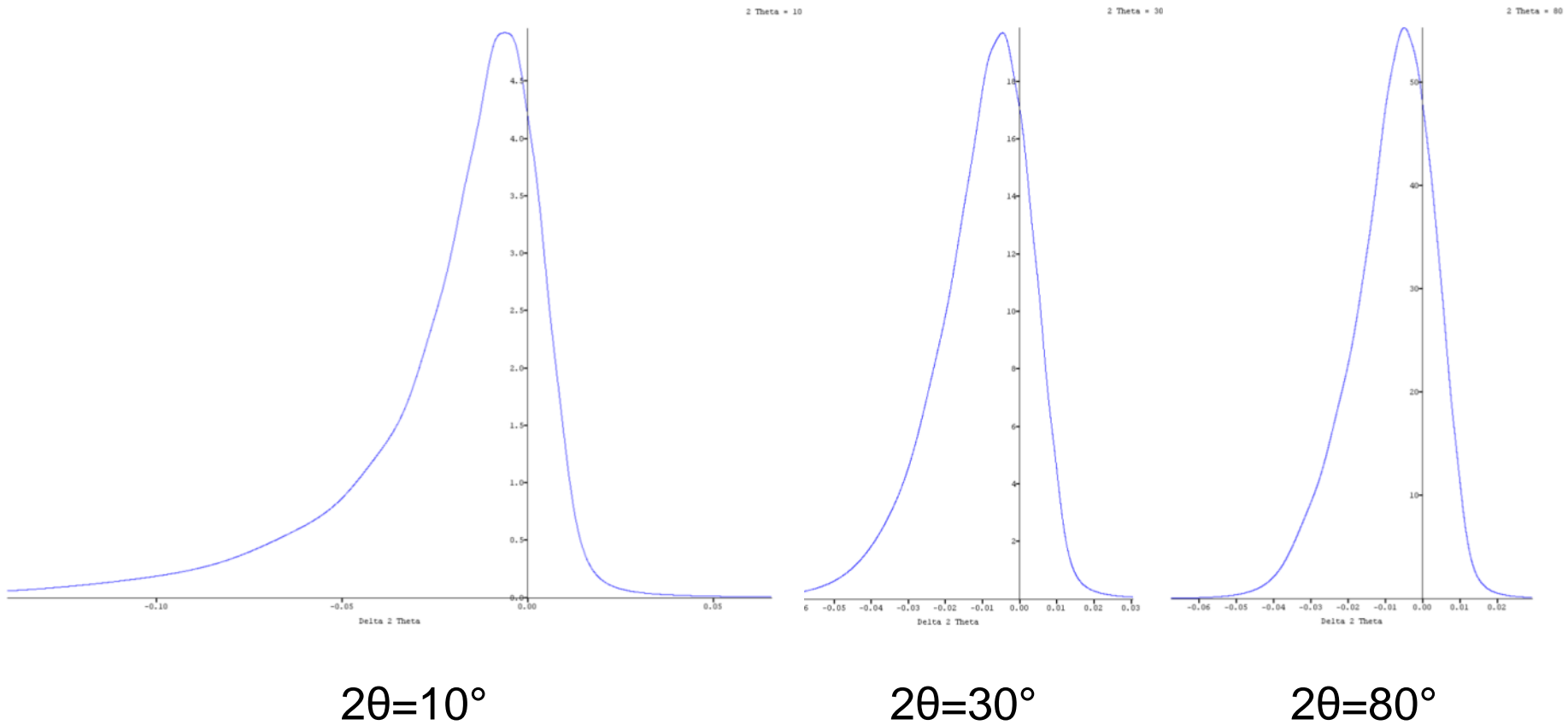
www.bruker.com

Fundamental Parameters Approach



Fundamental Parameters Approach

Calculated Peak Profiles



Fundamental Parameters Approach

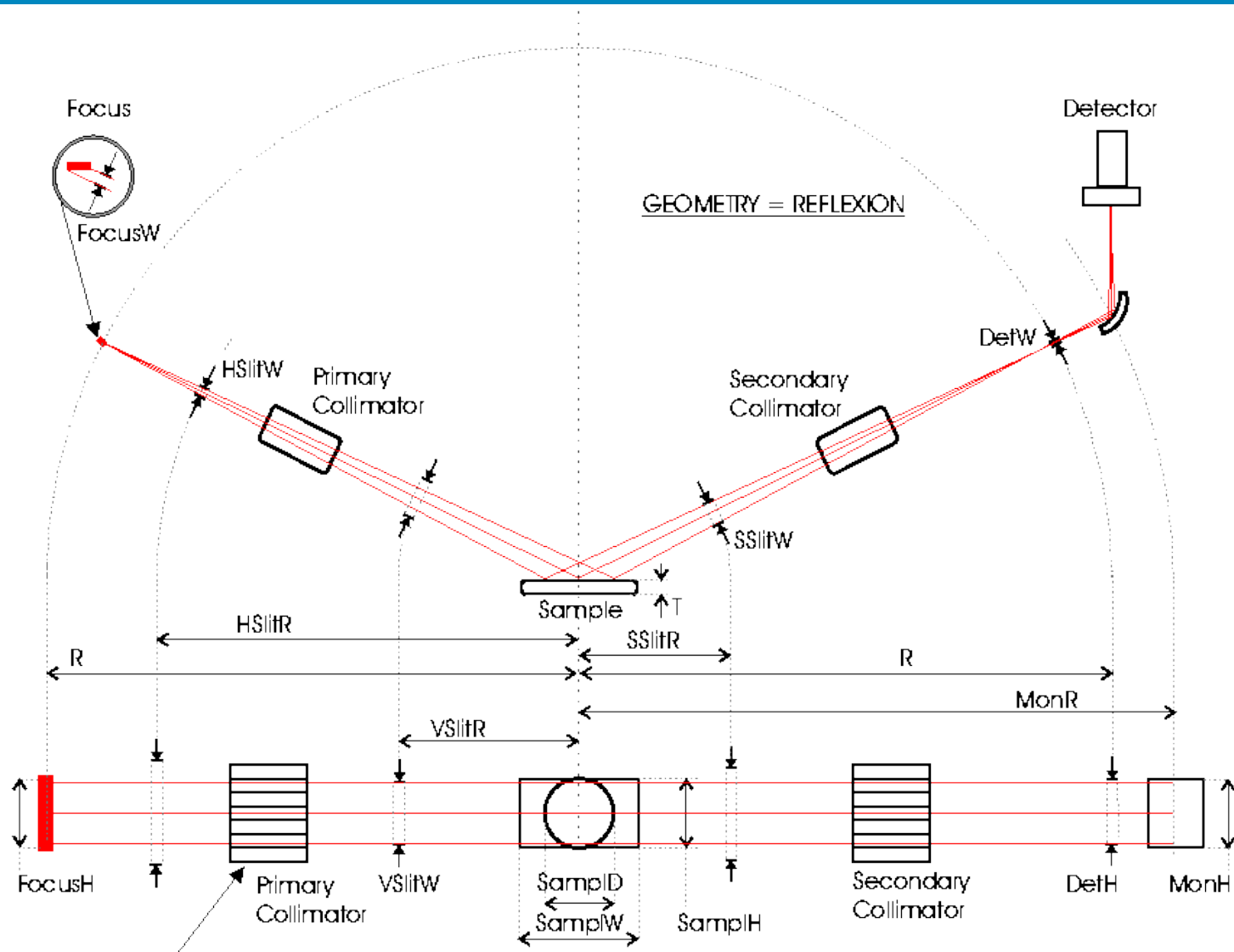
FPA needs:

- Very detailed and complete description of the instrument configuration
- Very well aligned instrument

- Some fundamental parameters are not documented
- Complete configuration can be hard to obtain

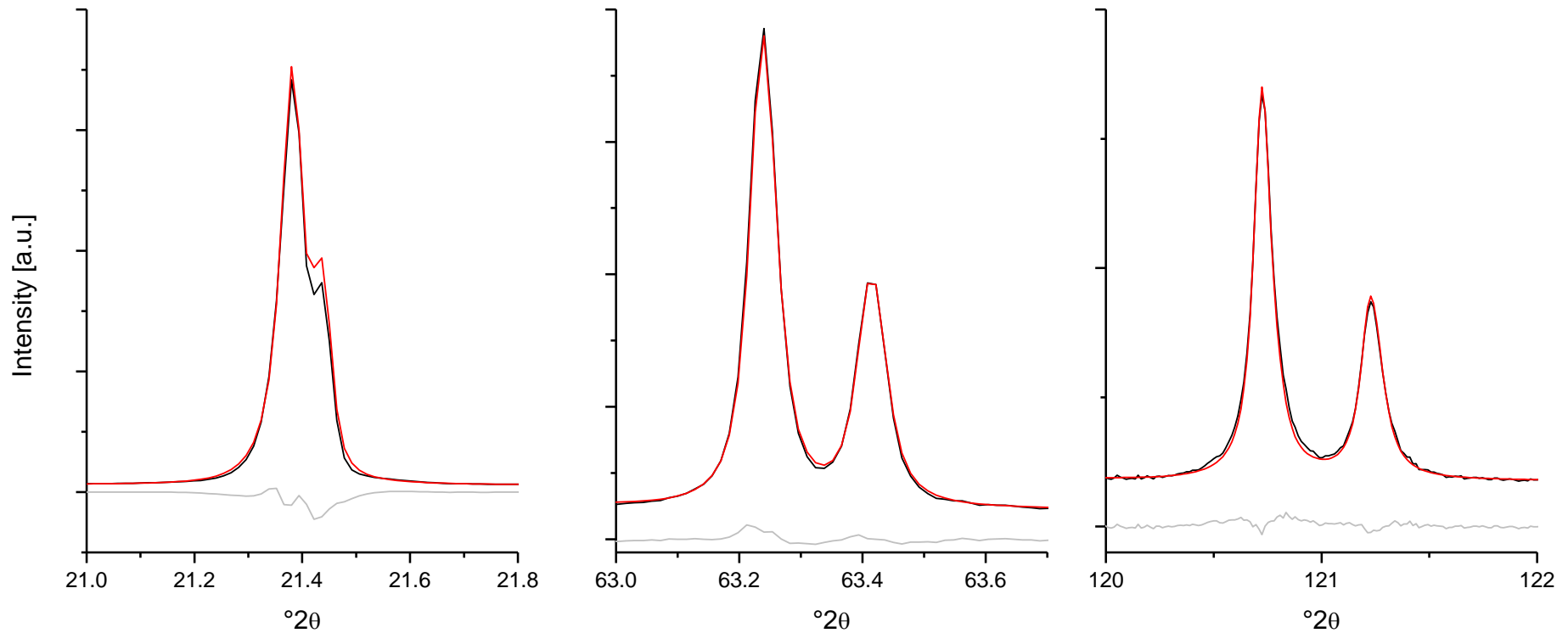


Fundamental Parameters Approach



Fundamental Parameters Approach

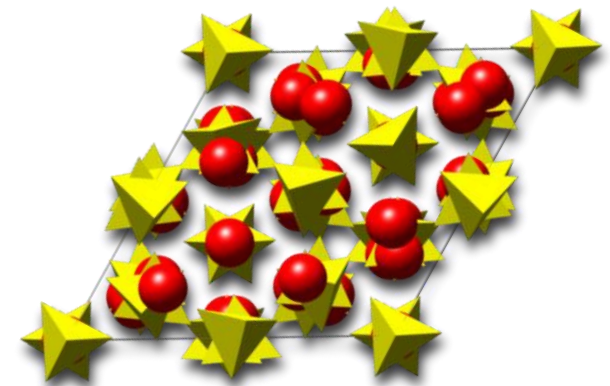
If done properly:



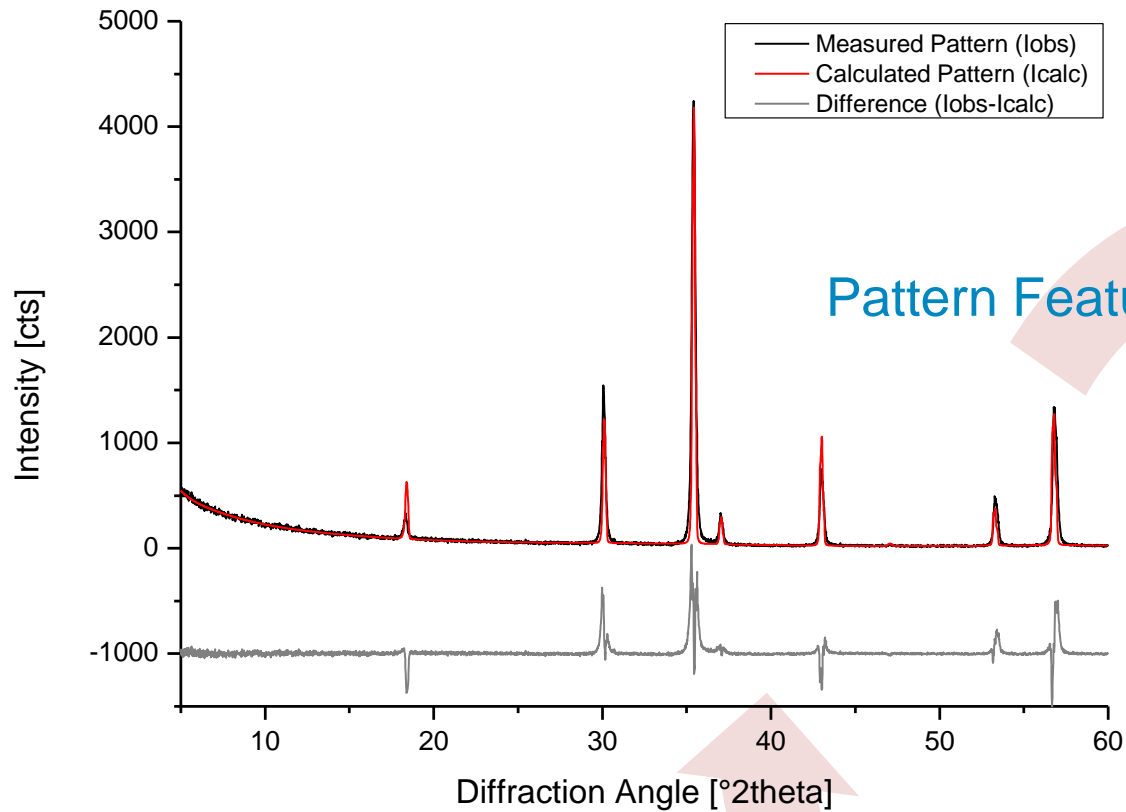
Very good description of the peak profile

Summary: Rietveld Basics

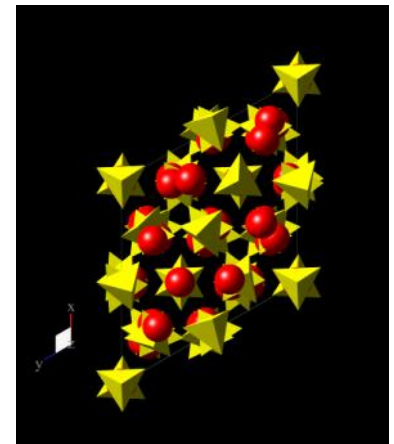
- Calculate XRD pattern from model structure
- Minimize differences between calculated and measured pattern
- Accurate mathematical description of peak profile required:
 - Classical Rietveld approach: Fit a peak shape function (PV or similar) to reference pattern
 - Fundamental Parameters Approach: Calculate peak profile from device configuration



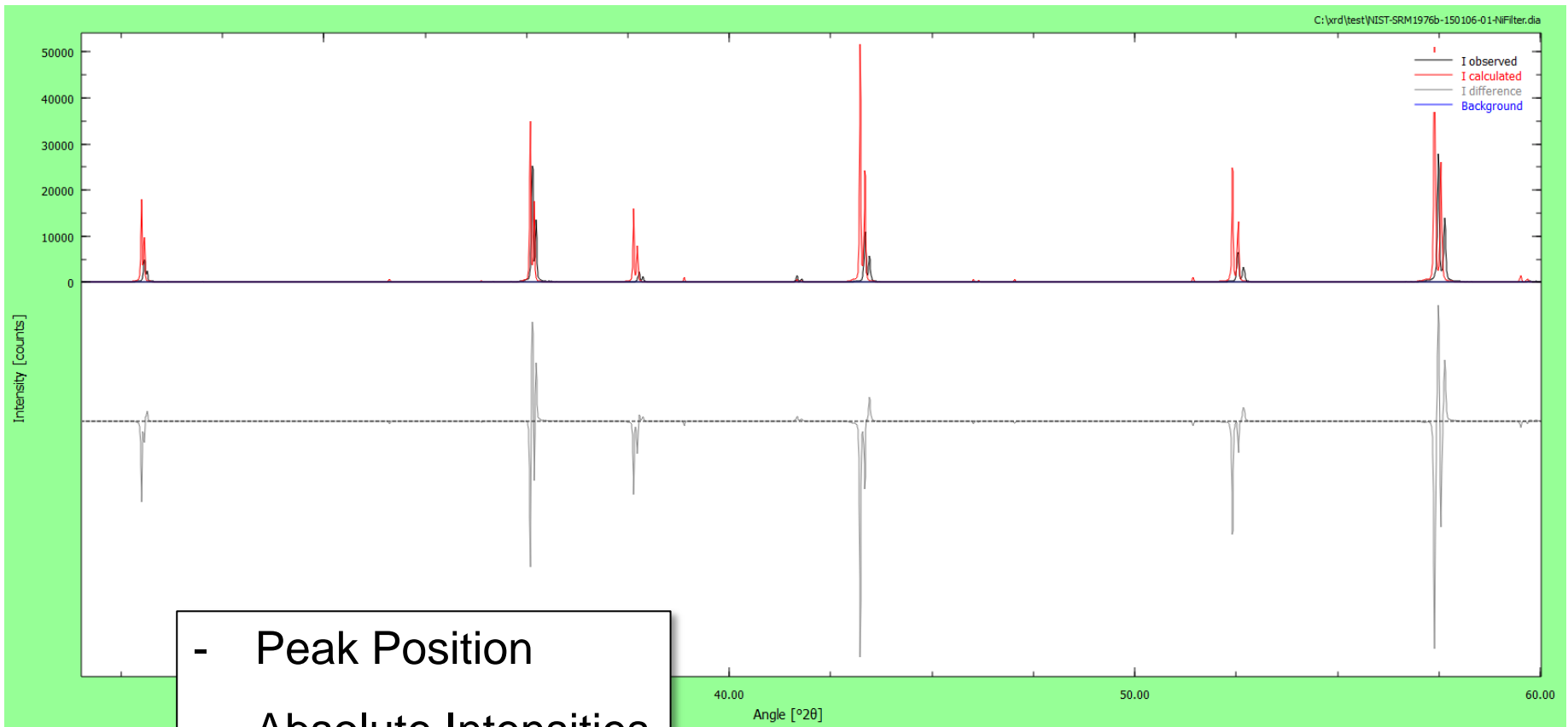
Refinement Strategies



Relation
Pattern Features – Structural Features



Refinement Strategy: Mismatches

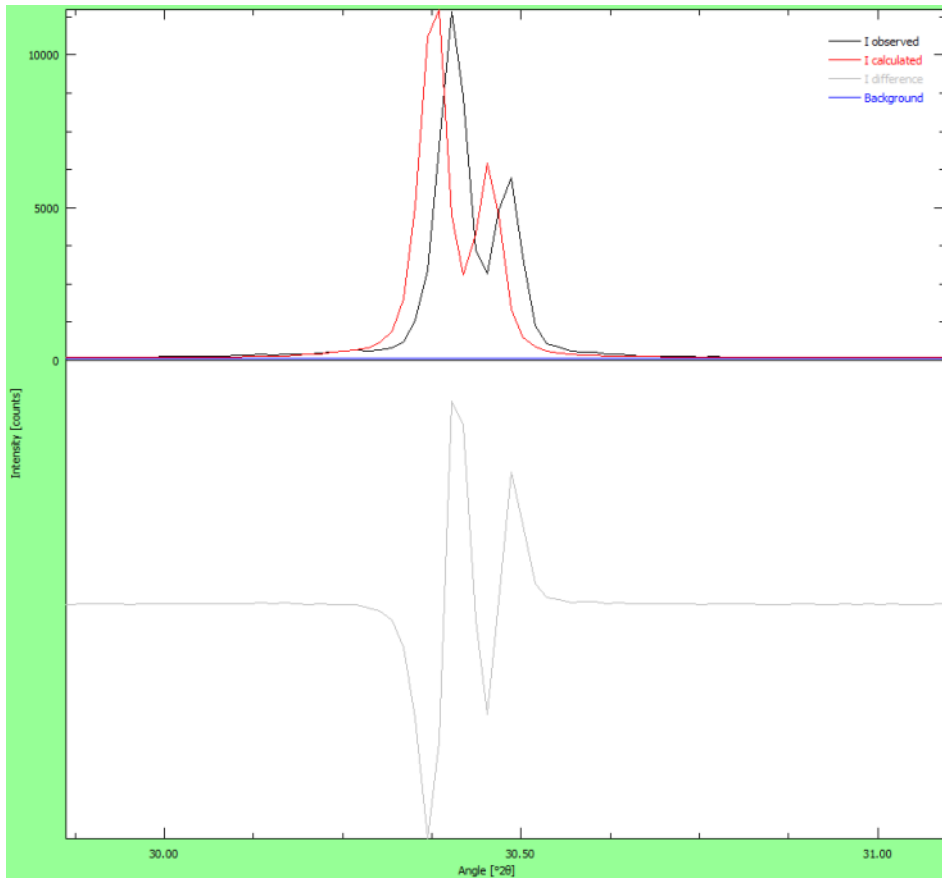


- Peak Position
- Absolute Intensities
- Relative Intensities
- Peak Width

How to fix this?



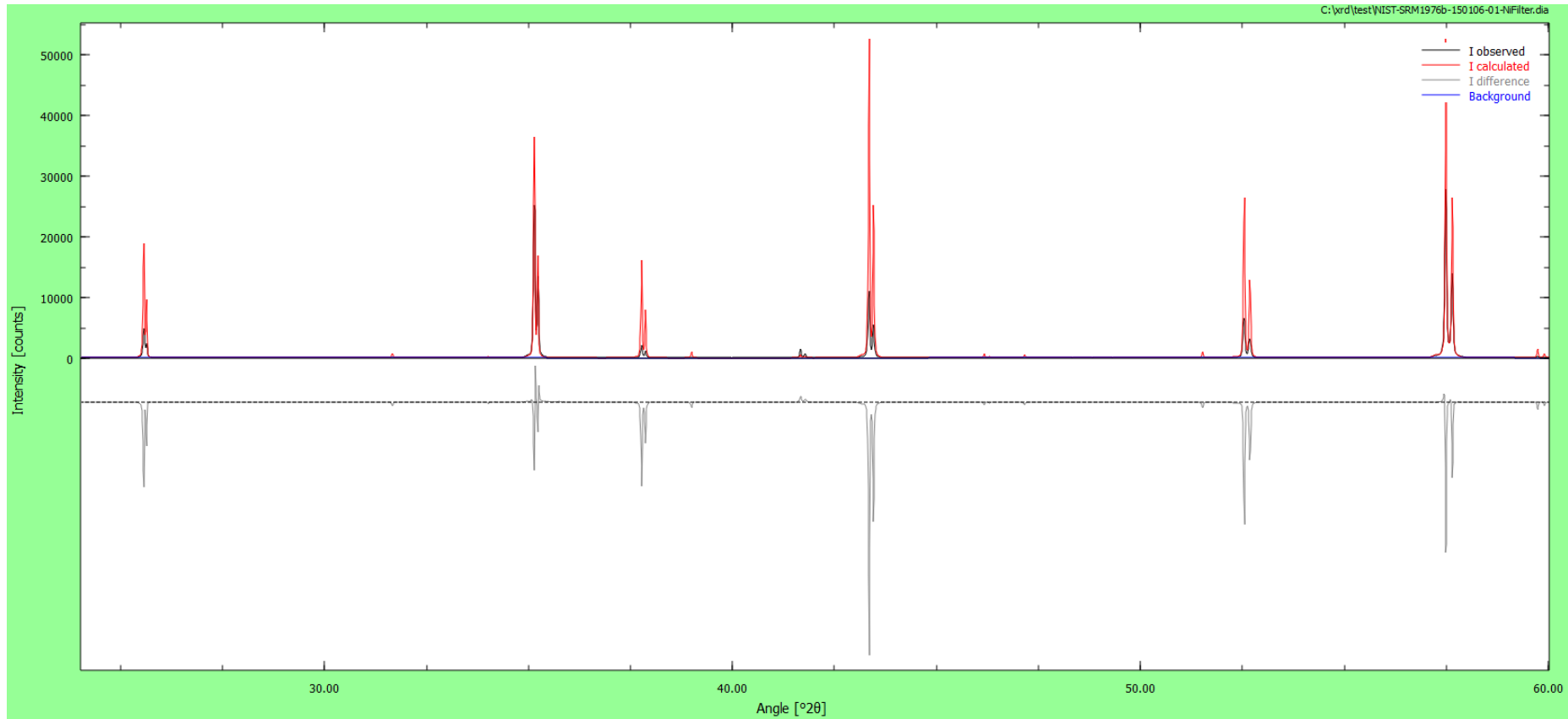
Refinement Strategies



Wrong peak positions:

- Unit cell dimensions
- Sample height displacement
- Zero-shift (instrument misalignment)

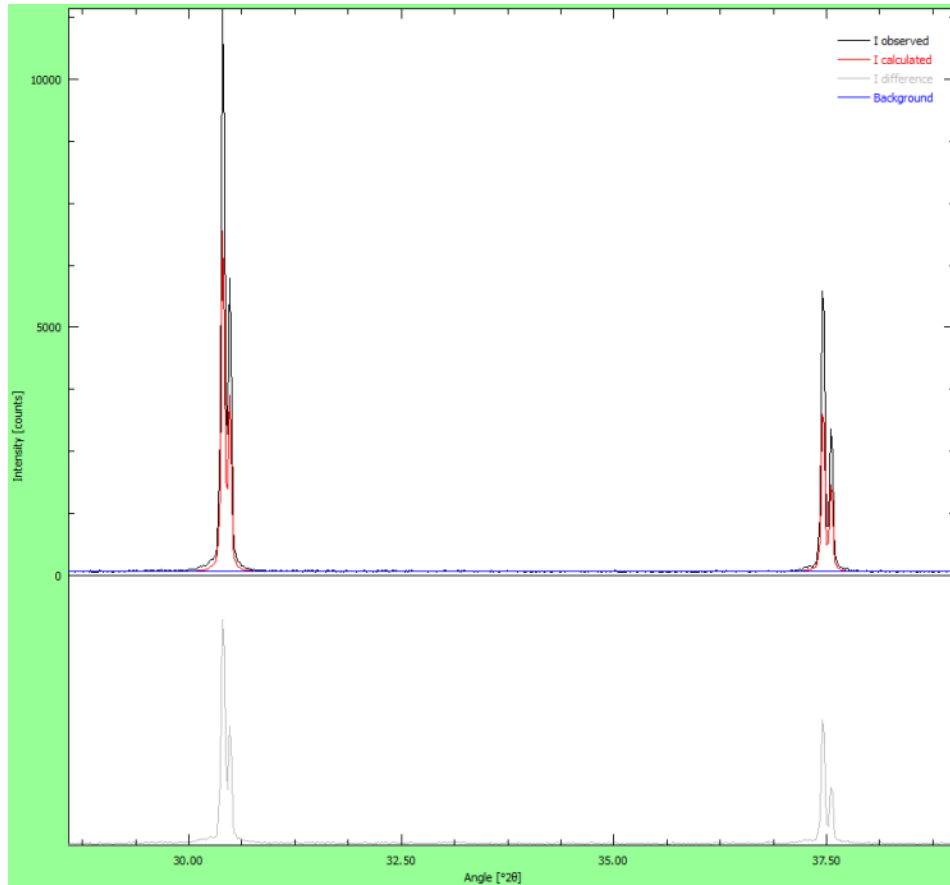
Refinement Strategies



Refined unit cell dimensions:

Peak positions matched!

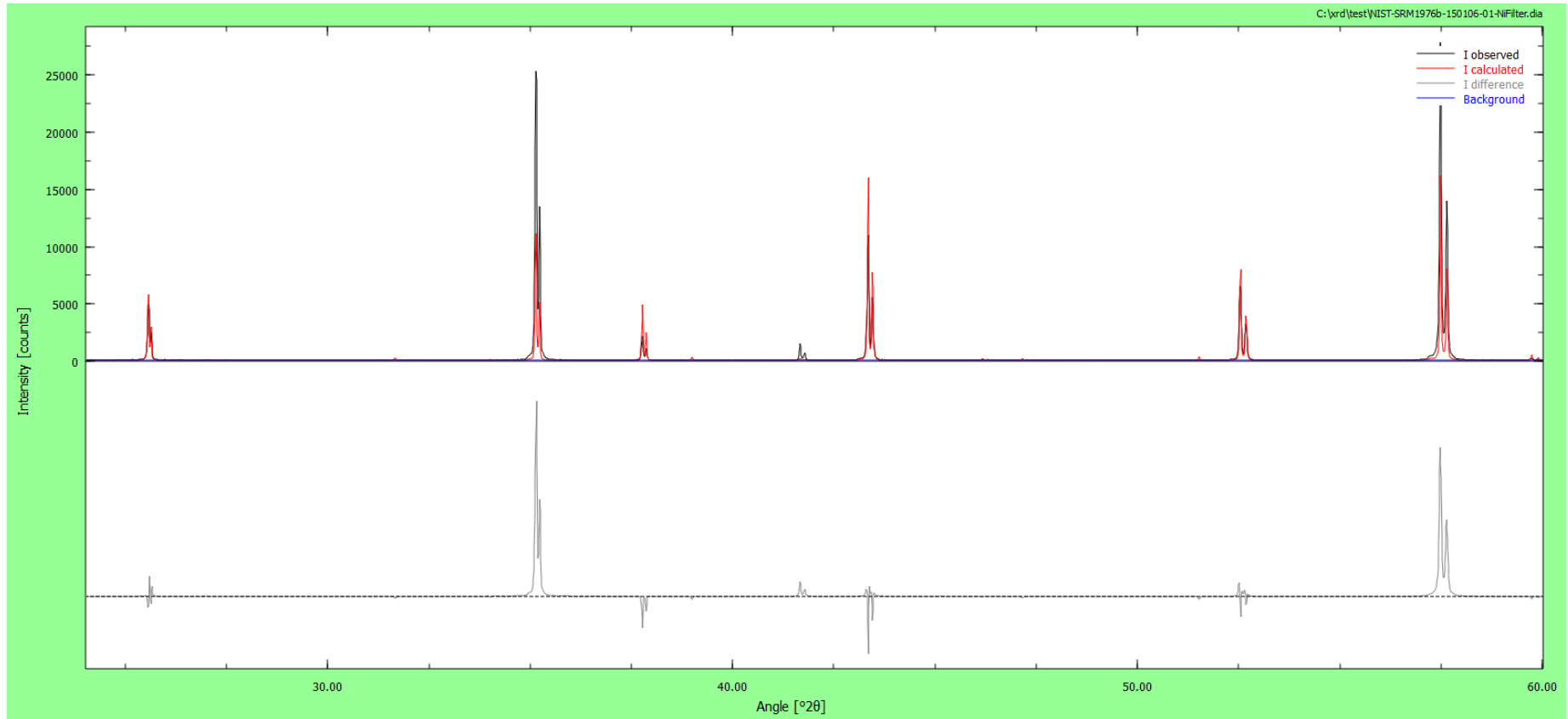
Refinement Strategies



Wrong absolute intensities:

- Weight fraction (scaling)

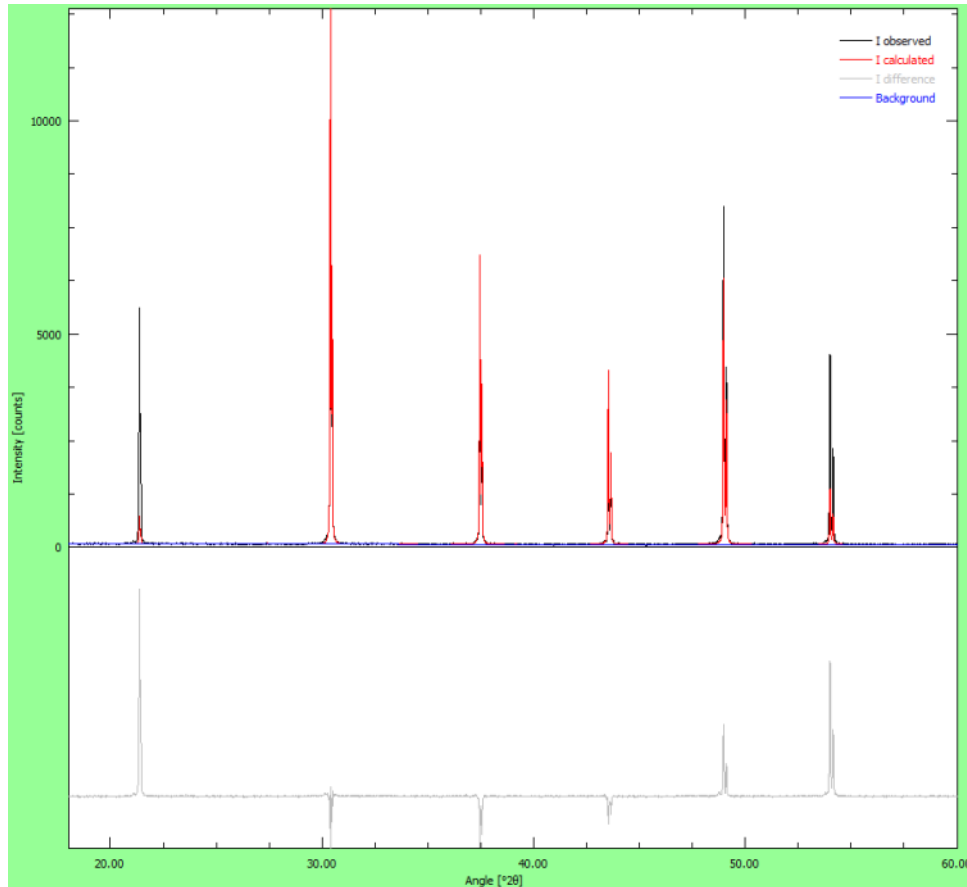
Refinement Strategies



Refined scale factor:

Intensities improved (but not fixed)!

Refinement Strategies

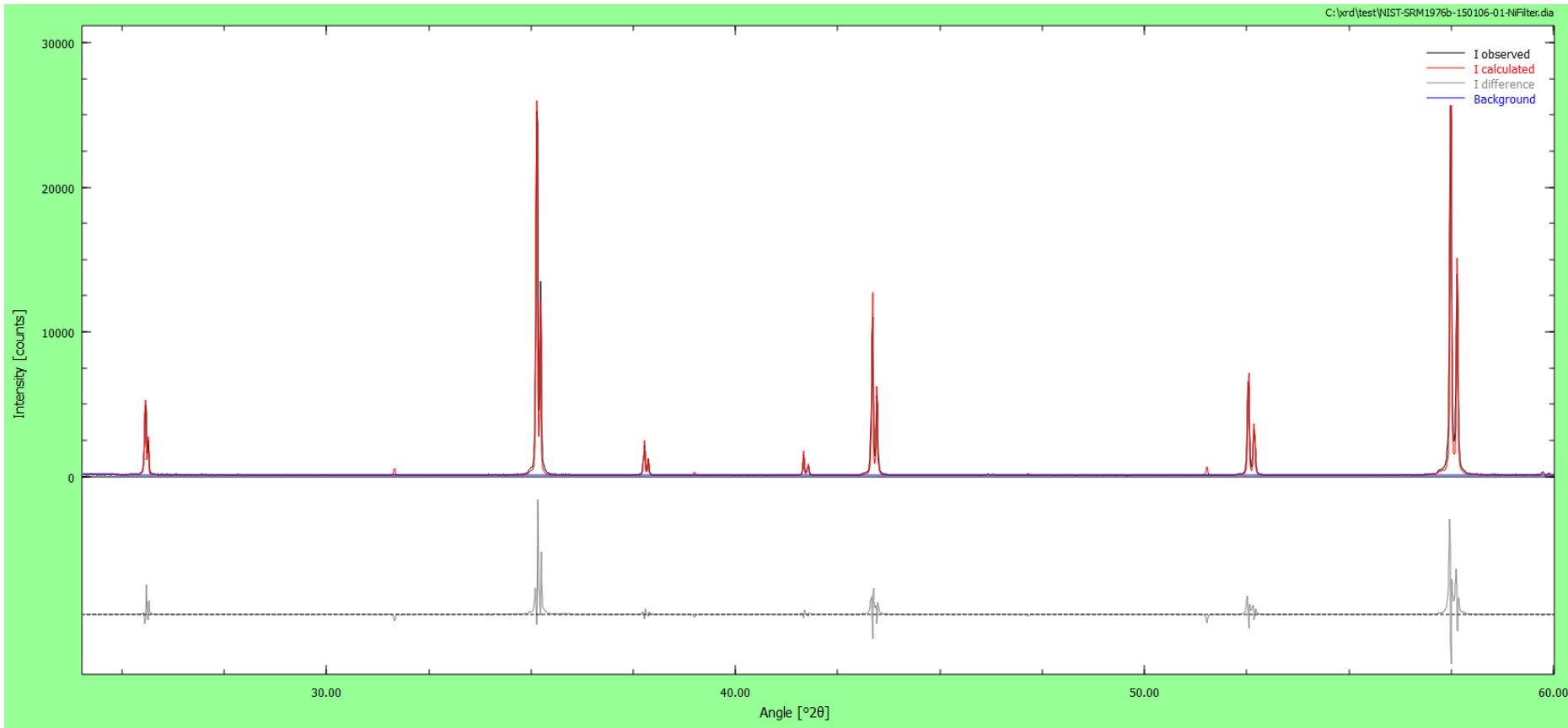


Wrong relative intensities:

Let's try this first

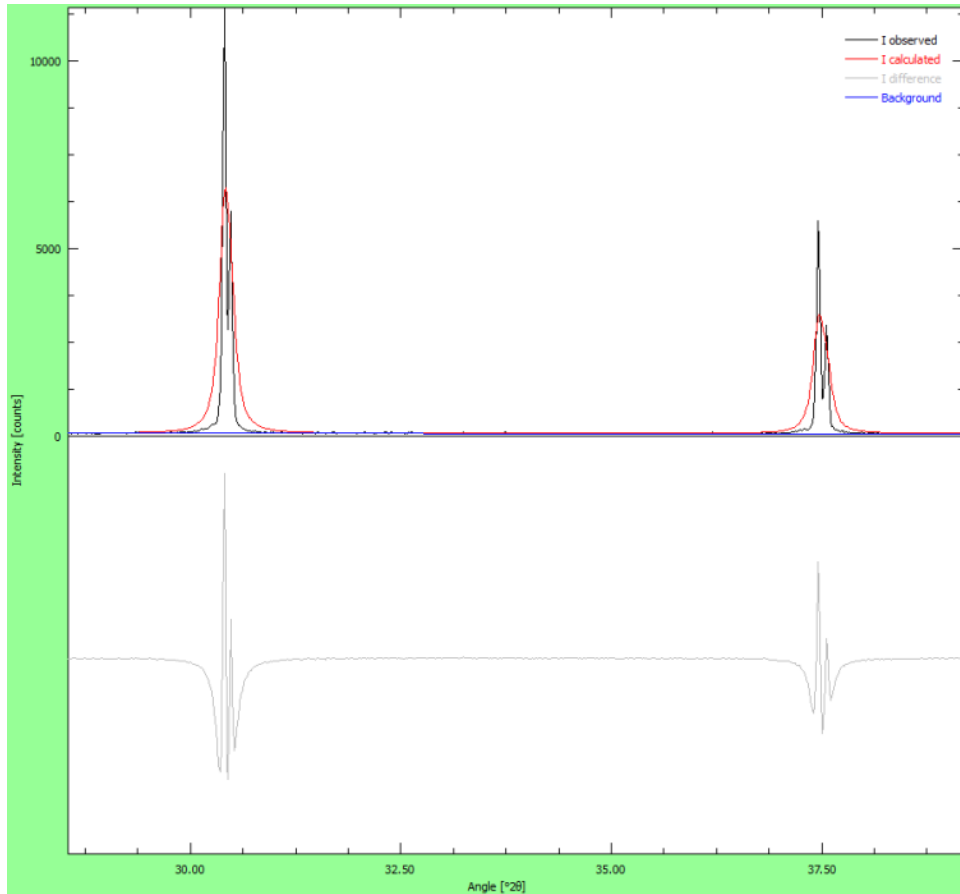
- Preferred orientation
- Graininess
- Atomic species
- Atomic coordinates
- Site occupancies
- Thermal displacement parameters

Refinement Strategies



Refined texture:
Intensities fixed!

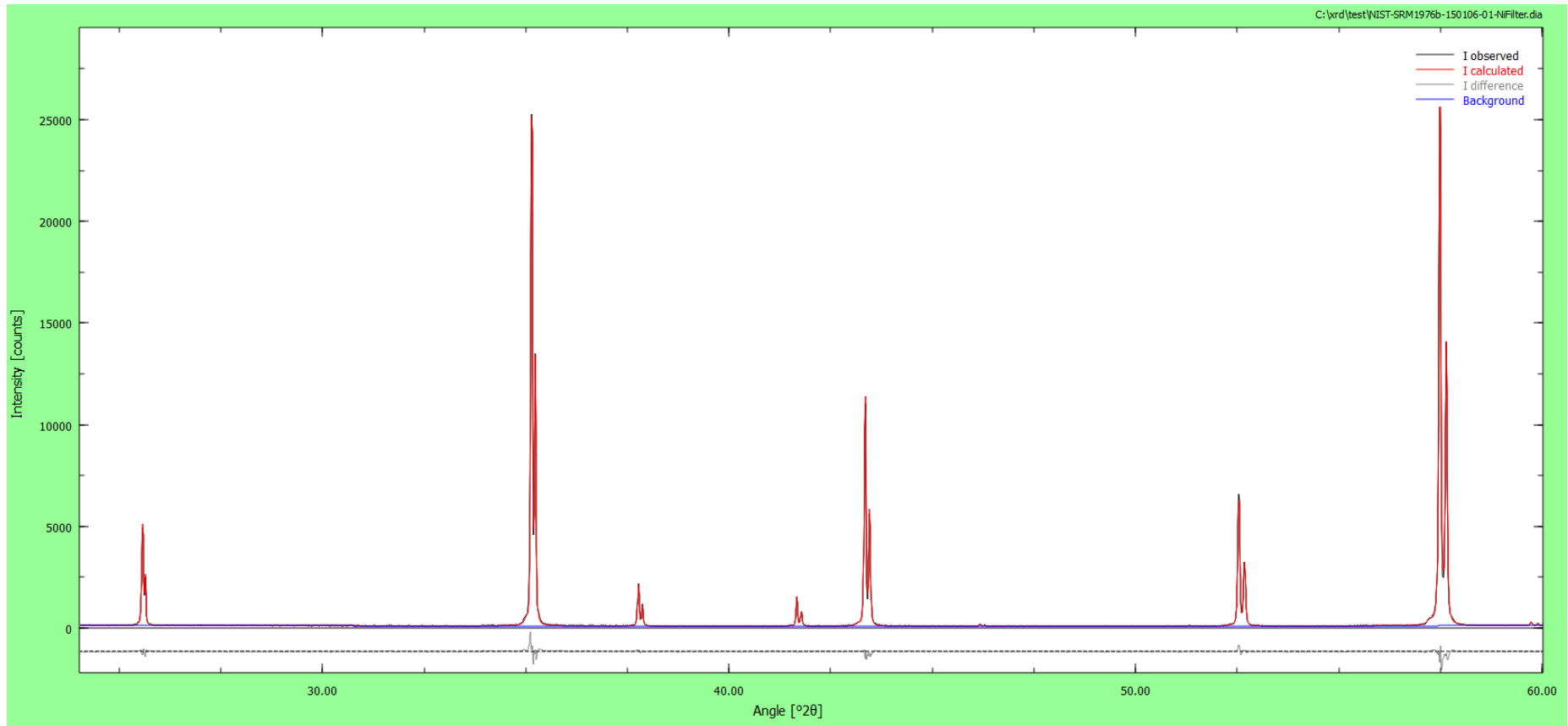
Refinement Strategies



Wrong peak width:

- Crystallite size
- Micro-strain in crystal structure
- Surface roughness

Refinement Strategies



Refined crystallite sizes and micro-strain:

Peak shape fixed!

Refined Crystal Structure

Phase composition: 100% Al₂O₃ Corundum

Starting Model

Parameter	Value
Unit cell <i>a</i>	0.4775 nm
Unit cell <i>c</i>	1.2993 nm
Crystallite Size	Inf.
Atomic Coordinates Al	0.0 / 0.0 / 0.3522
Atomic Coordinates O	0.3062 / 0.0 / 0.25

Refined

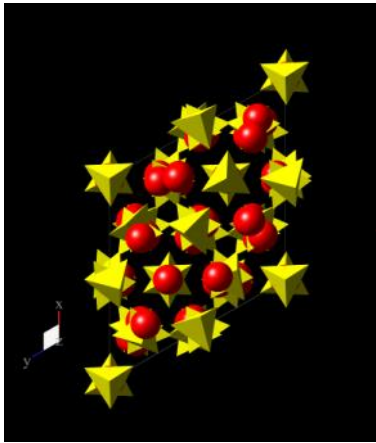
Parameter	Value
Unit cell <i>a</i>	0.4760127 +- 0.0000028 nm
Unit cell <i>c</i>	1.2995974 +- 0.0000077 nm
Crystallite Size	1267 +- 138 nm
Atomic Coordinates Al	0.0 / 0.0 / 0.3522
Atomic Coordinates O	0.3062 / 0.0 / 0.25

Summary: Refinement Strategy

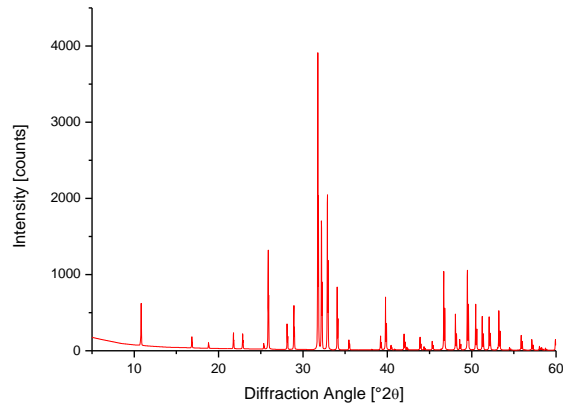
Effect in diffraction pattern	Origin in crystal structure model
Wrong peak positions	Unit cell dimensions Sample height displacement Zero-shift
Wrong absolute intensities	Weight fraction (scaling)
Wrong relative intensities	Preferred orientation Grainy sample Atomic species / Substitutions / Vacancies Atomic coordinates Site occupancies Thermal displacement parameters
Wrong peak width	Crystallite size Micro-strain Surface roughness Transparency

Rietveld Refinement

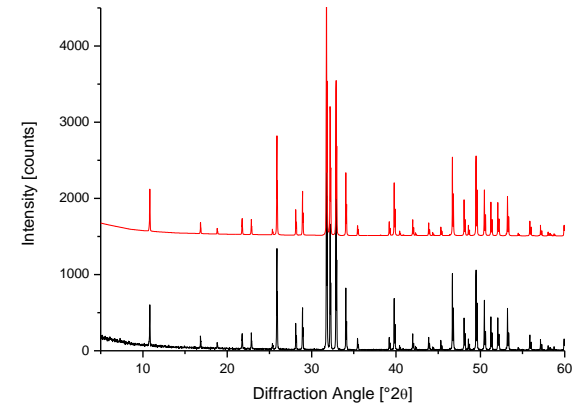
Known structure model



Calculate theoretical diffraction pattern



Compare with measured pattern



Optimize structure model, repeat calculation



Minimize differences between calculated and observed pattern by least-squares method

Rietveld Software Packages

Academic Software:

- Fullprof
- GSAS
- BGMN
- Maud
- Brass
- ... many more¹⁾

FPA

Commercial Software:

- HighScore+ (PANalytical)
- Topas (Bruker)
- Autoquan (GE)
- PDXL (Rigaku)
- Jade (MDI)
- WinX^{POW} (Stoe)

Commercial UI
for BGMN

Lesson 6: BGMN and Profex

1) http://www.ccp14.ac.uk/solution/rietveld_software/index.html